



PROCEDURE FOR ASSESSING THE DISPLACEMENT DUCTILITY BASED ON SEISMIC COLLAPSE THRESHOLD AND DISSIPATED ENERGY BALANCE

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Abstract

Displacement ductility is a parameter that characterizes the seismic response of structures. Moreover, displacement ductility can be used in order to determine whether a structural design, performed according to a specific seismic code or not, may achieve the main goal of the seismic design: to develop energy dissipation in a stable manner. There is unanimous agreement among the scientific community that the determination of the displacement ductility is not an easy task, because the structural response usually does not show a clear location of the points that define yield and ultimate displacements, necessary for its calculation. In this paper, the main procedures for ductility displacement are revised and compared, and then improvements are performed to such procedures to compute the displacement ductility. Improvements lead to determine the ultimate displacement using the seismic collapse threshold and the yield displacement by means of the balance of dissipated energy.

The procedure has been used to calculate displacement ductility of reinforced concrete framed buildings with different kind of non-linear response (designed with and without seismic code prescriptions). Results depict that the procedure is suitably for the objectively determination of the displacement ductility of buildings with column-sway, mixed or beam-sway failure mechanisms.

Keywords: Displacement ductility; dissipated energy; ultimate displacement; non-linear analysis; response reduction factor.

1. Introduction

Displacement ductility is an important characteristic that lead to determine whether seismic response of structures meet the early goals for which they were designed. Displacement ductility is also important because some of the most relevant seismic codes worldwide prescribe modal-spectral analysis for which the response reduction factors are estimated based on displacement ductility [1]. The relevance of this work was the identification of the components that define the response reduction factor, depending on three factors:

$$R = R_{\mu} \cdot \Omega_d \cdot R_R \quad (1)$$

where R_{μ} , Ω_d and R_R are the ductility reduction factor, the overstrength factor and the redundancy, respectively. There are several procedures to determine values of displacement ductility, but each produce values with high variability, then does not exist consensus among engineers and researchers about how to choose suitable values of displacement ductility for design [2, 3, 4].

The concept used to determine displacement ductility is too simple: to establish any relationship between the elastic and plastic behavior through specific displacements. Since Freeman [5], scientific and engineering community have accepted the pseudo-static analysis procedure with lateral forces, also called Pushover analysis.

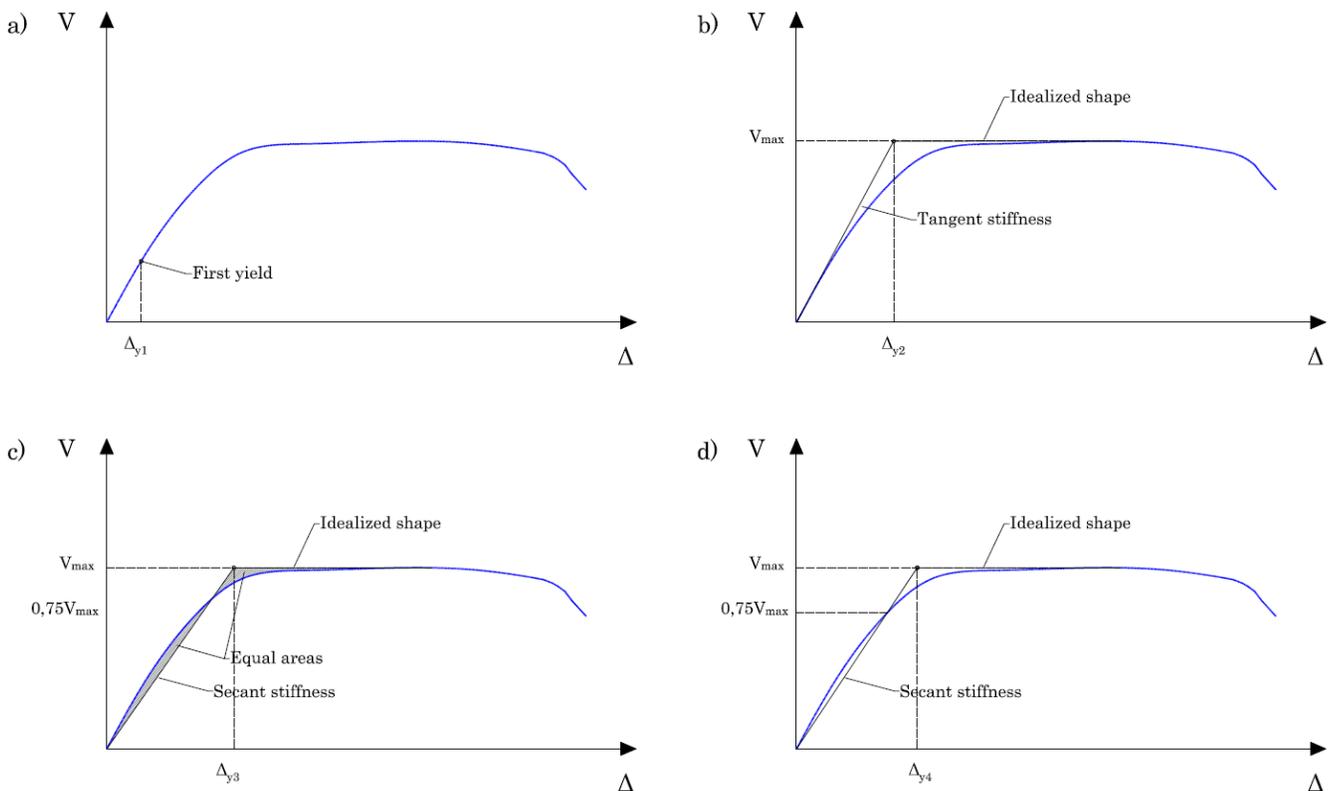


Fig. 1- Alternatives for global yield displacement determination, adapted from [6]

The problem with the definition of displacement ductility arise when the components must be defined. First, there are some ways to determine the yield displacement of the whole structure, see Figure 1 (adapted from [6]). Fig.1a show the definition of yield displacement at the point of occurrence of the first yield in any of the reinforcement bars of the structure. Fig.1b show the criteria based on the tangent stiffness, defined by the elastic stiffness. In Fig.1c, the yield displacement is obtained from the secant stiffness, defined by equalizing the dissipated energy of the capacity curve and the idealized bi-linear shape. Finally, Fig.1d show the yield

displacement defined by Park [7]. Note that according to this procedure, the secant stiffness correspond to a line from the origin to a point on the capacity curve with 75% of the maximum base shear capacity.

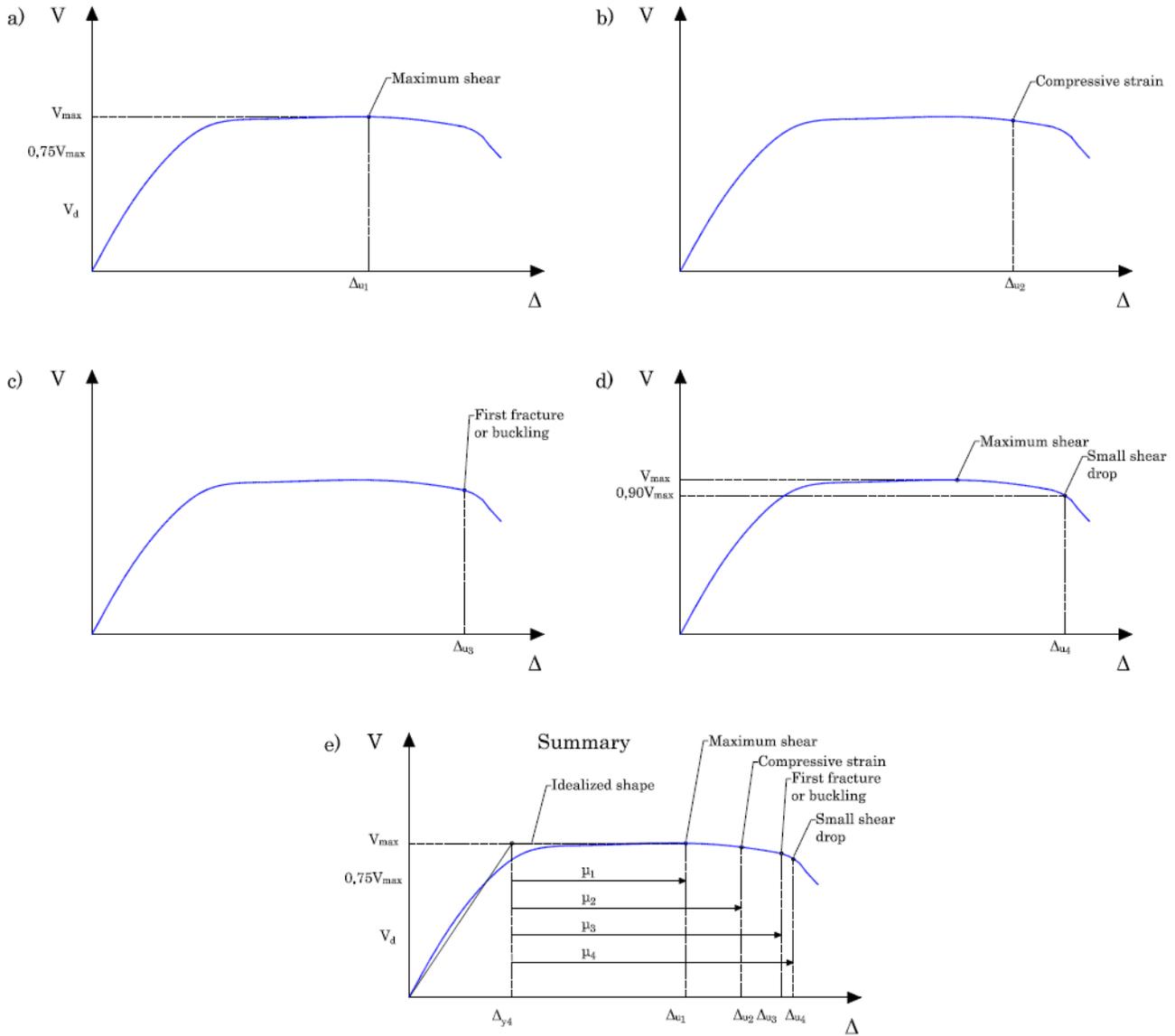


Fig. 2- Alternatives for global ultimate displacement determination, adapted from [6]

Determination of the ultimate displacement is also controversial. One of the most popular criterion is to assume that the ultimate displacement correspond to the point where the structure reaches the maximum base shear (Fig. 2a). Another used criterion to determine the ultimate displacement assumes that the ultimate displacement correspond to the point in which compressive strain in concrete or the fracture or buckling in transversal reinforcement in any structural member is achieved (Fig.2b and 2c). The criterion showed in Fig2d is also very popular. This criterion lead to obtain the ultimate displacement from the point in which the maximum base shear capacity drops a fixed value (usually 10 or 20%). Fig.2e summarizes ductility calculations based on the above-mentioned criteria. Note the difference among resulting values, consequently, the need to develop a procedure in order to objectively determine reliable displacement ductility factors. Similar comparison about the referred ultimate displacements can be found in [2].

2. Proposed procedure

The new procedure is based in some of the relevant features discussed in the introduction. The procedure to determine the displacement ductility based on non-linear analysis consist in three steps defined as follows:

First step. The first segment consist in a line from the origin to the point in which the structure presents the first yielding in any bar of the structural members. The latter point will serve as a “pivot” point to find the yield displacement.

Second step. Determine the ultimate displacement. To this end, it is necessary to find the base shear for which ultimate rotation capacity in the extremes of the beams and the inferior extremes of the columns of any level are achieved.

Third step. Obtain the yield displacement equalizing the areas under the capacity curve and the generated tri-linear idealized shape. This step is performed by means of a couple of curves obtained from the integration of the capacity curve and the tri-linear idealized shape. The point where these two curves intersect, is compared with the point for the ultimate rotation displacement, if the difference between the abscissas of both points was lower than 1%, the chose value of the abscissa for the yield point is accepted, on the contrary, the yield point must move horizontally in order to adjust the dissipated energy. This way leads to obtain a balance of the dissipated energy of the capacity curve and the idealized bi-linear shape, see Fig. 3.

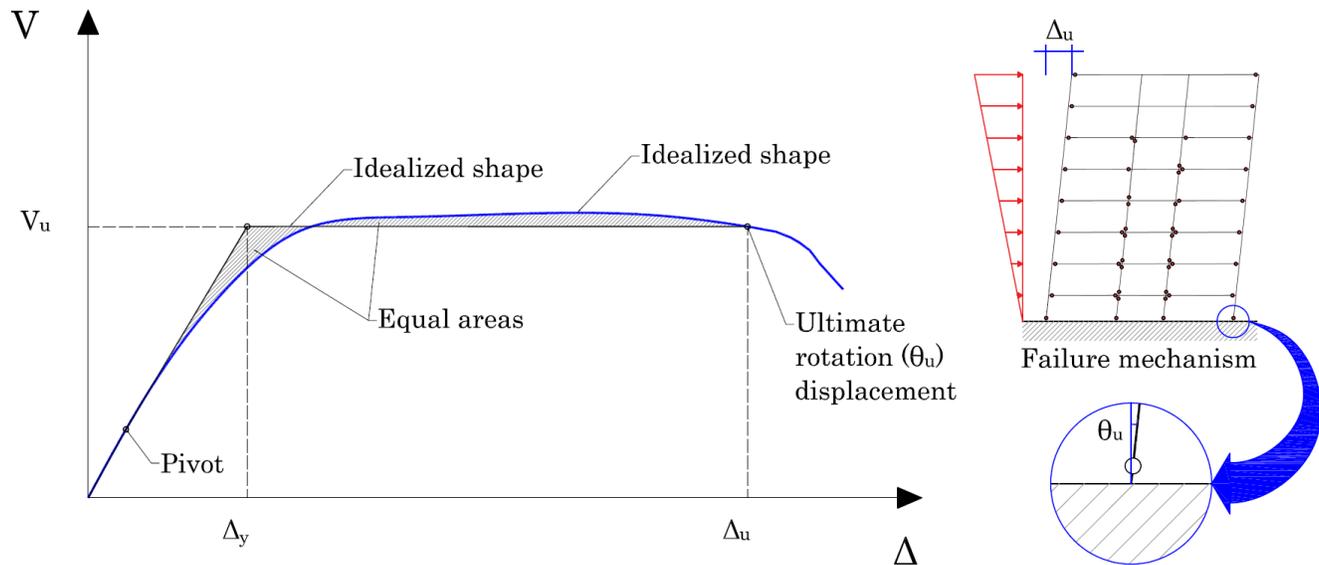


Fig. 3- Description of the objective procedure for displacement ductility determination

Determination of the seismic collapse threshold or the ultimate displacement is performed following the failure mode of the structure analyzed. For this purpose, the adopted criterion is that the whole structure has been reached its ultimate displacement when all the extreme elements of the beams and the lower elements in columns of any story achieve a rotation greater than the ultimate rotation capacity, defined by according to [8]:

$$\theta_u = \alpha_{st}(0,3^v) \left[\frac{\max(0,01,w')}{\max(0,01,w)} f_c \right]^{0,2} \left(\frac{Ls}{h} \right)^{0,425} 25^{(\alpha_{psx} f_{yw}/f_c)} \quad (2)$$

where: $\frac{Ls}{h} = \frac{M}{Vh}$: Moment-shear ratio at the member end

w, w' : Mechanic reinforcement ratio

ν : Normalized axial load ratio

ρ_{sx} : Ratio of transverse steel

α : Factor confinement effectiveness factor

The procedure is carried out by assigning an ultimate chord rotation to each structural element, according to the results of Equation (2). In this way, for every increment of applied lateral load, computed chord rotations were compared versus the assigned values of ultimate rotation capacity, indicating whether an element has achieved its ultimate rotation [9], see Fig. 4. When all elements located at the ends of the beams belonging to one story and the lower ends of the columns of the immediately below story, reach their ultimate chord rotations, then, it is assumed that the structure is not be able to sustain additional lateral forces, so it has been reached the collapse threshold displacement. Then, the ultimate base shear corresponds to the sum of the reactions in supports opposing to lateral forces in the indicated load step.



Fig. 4- Load step for the collapse, controlled by the ultimate rotations at the end of first-floor beams and columns (ductile failure)

3. Cases studied

The relevance of formulating a new procedure to obtain the displacement ductility resides into capture an objective value, regardless of the possibility to reach a fragile, mixed or ductile structural failure mode [10]. In order to test the procedure, it was applied to a set of buildings with different plan configurations. The set consist into seven low-rise reinforced concrete buildings, designed for high seismic hazard zone (design acceleration of 0.3g) and located in very stiff soil (soil type S2) with a response reduction factor $R=6$, see the elastic and inelastic design spectra in Fig.5.

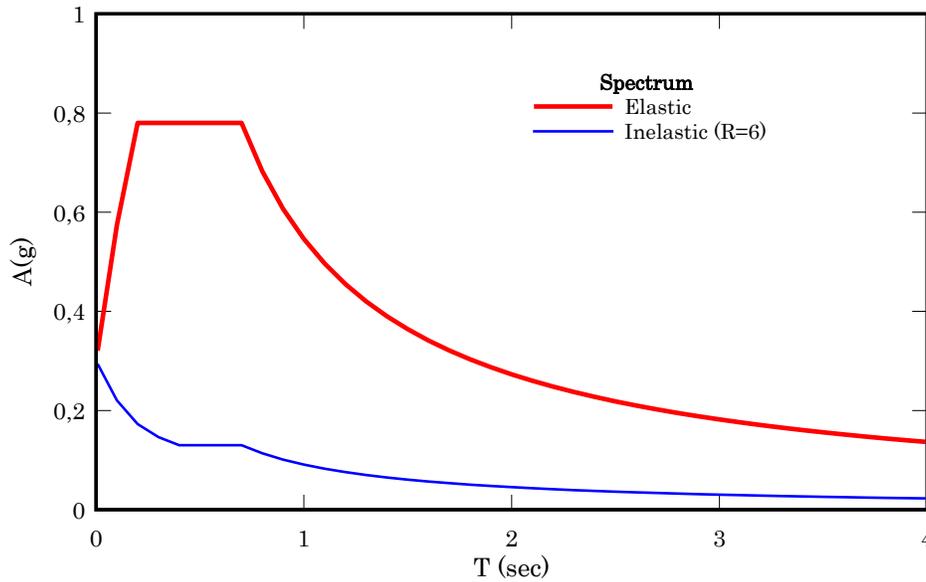


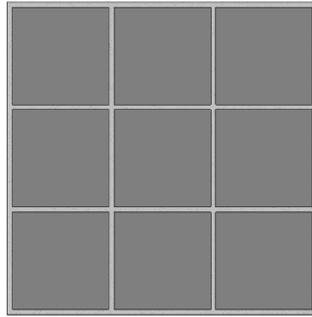
Fig. 5- Elastic and inelastic design spectra

Buildings have different plan configurations with three 3.00m high stories. The structures of the buildings consist in special moment-resisting frames, with three 6.00m length spans in each direction. These frames bear 25cm width RC solid slabs. Fig. 6 summarizes the plan configurations of buildings. Note that cases 2 to 7 are plan irregular because the presence re-entrants in the slabs, but cases 2, 4 and 6 are provided with continuous beams in the open side, avoiding the loss of stiffness in such frames, also avoiding the stress concentration in such frames and adjacent zones which can occur during the application of lateral loads. The specifications set for the materials are: concrete $f'_c = 25MPa$ and steel reinforcement $F_y = 420MPa$.

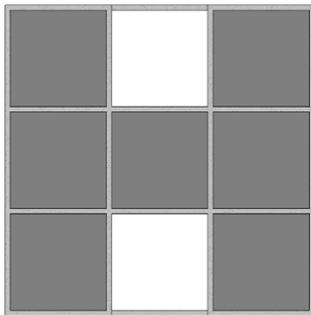
The analysis, design and detailing of the buildings was performed according to the current Venezuelan seismic code [11] for residential use only. However, interstory drift check was performed using an alternative energy-based procedure [12, 13]. Resulting structures were modeled using v-7 of SeismoStruct software [14]. Resulting RC cross-sections were modeled using fibre elements, with the accurate location of every reinforcing bar, and taking into account the effect on the concrete of the confinement provided by transversal and longitudinal reinforcement bars.

Once the structures were modeled, standard pseudo-static non-linear analyses (Pushover) were performed, using a linear distribution shape for lateral forces, with a target roof displacement estimated as 4% of the total building high. The analysis were performed for both directions of the buildings, in order to account the influence of the plan irregularity in the capacity curve determination and in the damage distribution for each structural member. According to the criterion found in [2] no relocation of center of mass produced by accidental eccentricity was performed.

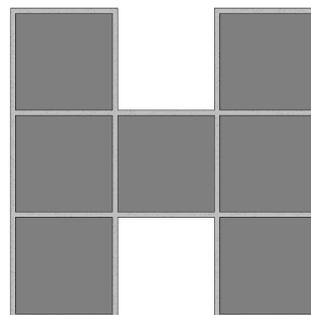
Case 1



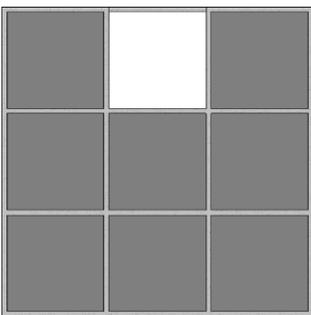
Case 2



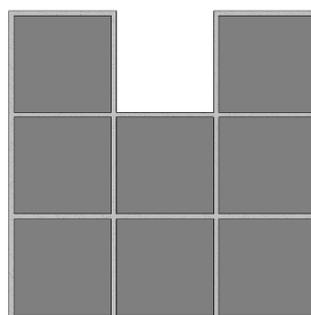
Case 3



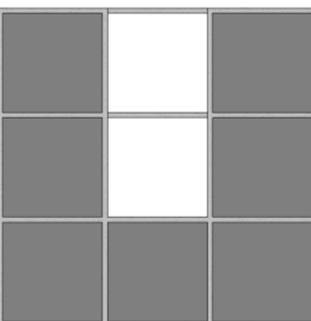
Case 4



Case 5



Case 6



Case 7

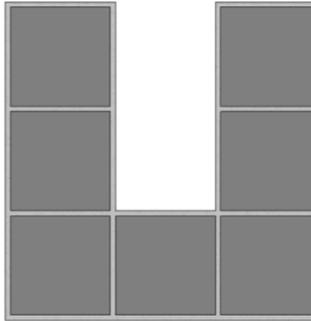


Fig. 6- Plan configurations of cases studied

4. Results

The capacity curves obtained for each building were processed according the Park’s procedure and the new procedure reported herein, in order to find the idealized shapes in both analysis directions and to determine the values of displacement ductility (μ) and overstrength (Ω_d). The values of both coefficients are summarized in Table 1.

Table 1- Values of displacement ductility and overstrength of cases studied

Case	Park’s procedure				Proposed procedure			
	x		y		x		y	
	μ	Ω_d	μ	Ω_d	μ	Ω_d	μ	Ω_d
1	2.89	3.46	2.89	3.46	5.57	3.36	5.57	3.36
2	2.77	3.92	2.70	3.87	5.37	3.84	4.90	3.79
3	3.43	3.52	3.25	3.99	5.38	3.49	5.49	3.92
4	2.95	3.71	2.98	3.66	6.74	3.58	5.27	3.58
5	3.16	3.41	3.04	3.70	5.97	3.34	5.19	3.63
6	2.91	3.98	3.10	3.91	5.20	3.90	5.21	3.85
7	3.47	3.49	2.98	3.99	6.64	3.39	5.88	3.91

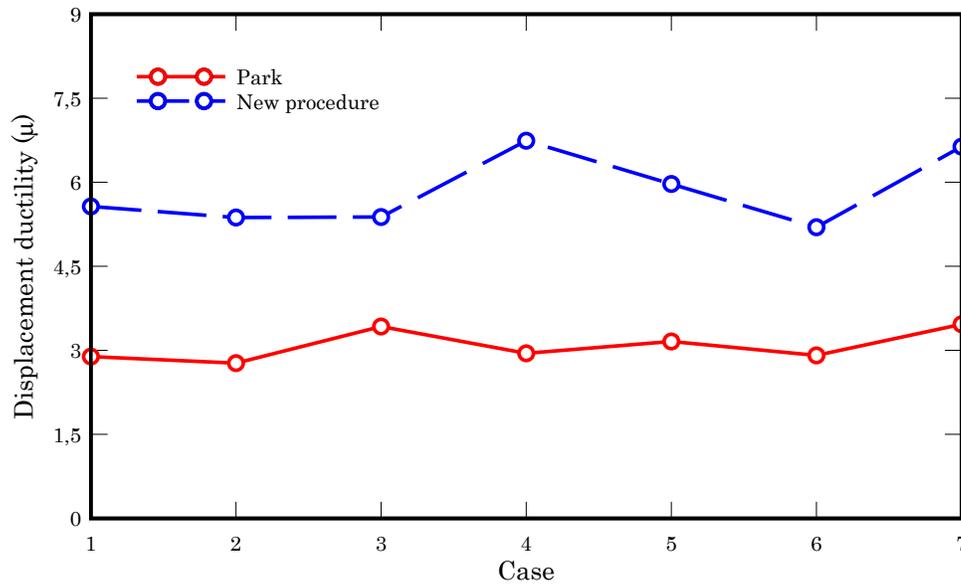


Fig. 7- Displacement ductility of cases studied, in x-direction

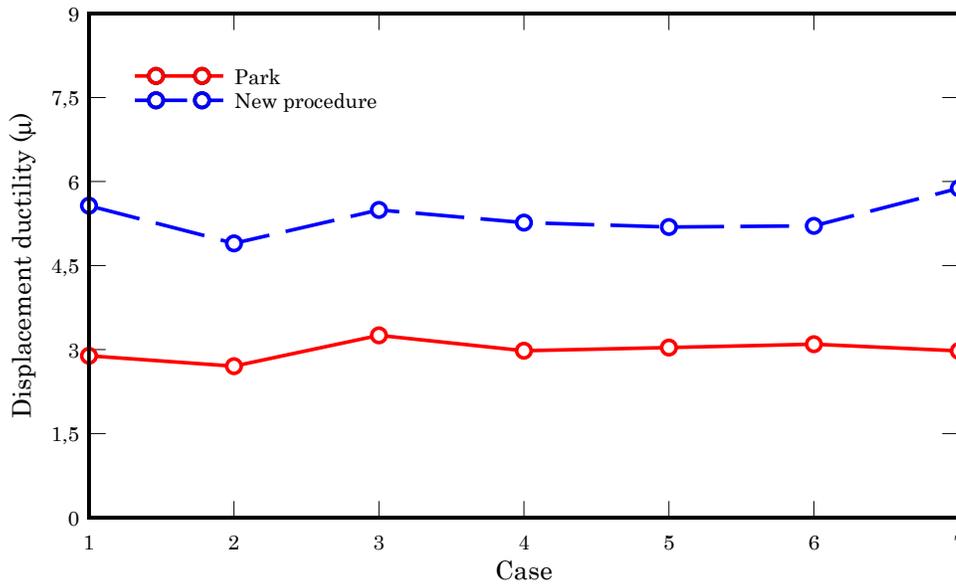


Fig. 8- Displacement ductility of cases studied, in y-direction

In general, calculated values of displacement ductility using the proposed procedure, show small variations, despite on the plan irregularity of the buildings. For analysis performed in the irregular direction of the cases (x direction) resulting values are less than 6, with the exception of cases 4 and 7, in which the values are near to 7, see Fig. 7. On the other hand, Fig. 8 displays the values of displacement ductility calculated for the regular direction of the cases (y direction). It can be observed the uniformity of the displacement ductility values, which are slightly less than 6.

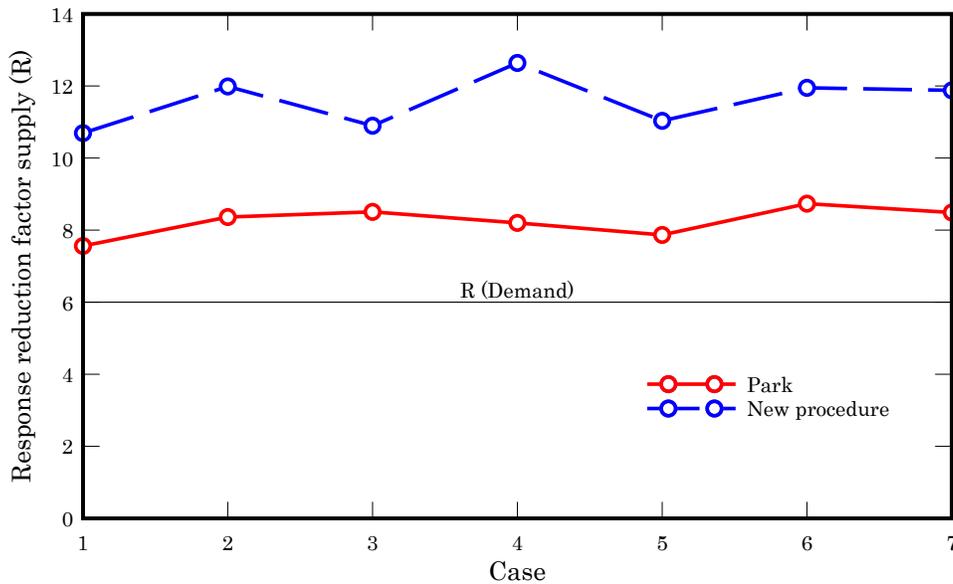


Fig. 9- Response reduction factors (supply) computed using Park's procedure vs. proposed procedure in x -direction

Values obtained using the proposed procedure are consistent with those expected for the used structural typology combined with the seismic hazard level. Furthermore, the use of the new procedure may serve for the calculation of the inherent response reduction factor of the cases studied according to Eq. (1). The inherent response reduction factor, also called response reduction factor supply [6], may serve as a reference value in order to evaluate the seismic design obtained through the response reduction factor prescribed by the seismic code (also called response reduction factor demand [6]).

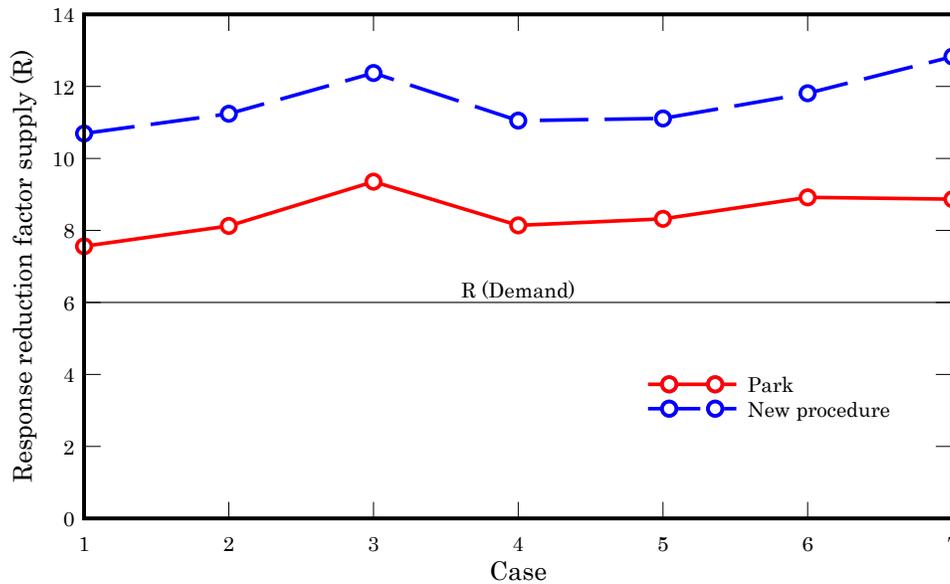


Fig. 10- Response reduction factors (supply) computed using Park’s procedure vs. proposed procedure in y-direction

Although both procedures used in this paper have produced values of the response reduction factor (R_{Supply}) which are greater than the R value prescribed by code (R_{Demand}), see Fig. 9 and 10, values calculated from proposed procedure are consistent with the values reported in recent works using incremental dynamic analysis (IDA), in framed structures designed in high seismic-prone regions [15, 16, 17, 18].

5. Conclusions

The proposed procedure combines the main goals of previous works in the field, with the end to obtain objective values of displacement ductility, regardless the structural type, failure mode or even the structural irregularities. This procedure is also simple, thereby enabling in displacement ductility determination using standard pseudo-static non-linear analysis.

Resulting values of the displacement ductility calculated according the proposed procedure are, in general, nearly uniform in the regular direction of case studied, and have values that lightly vary according the irregular direction of analysis. These values are greater than the ductility values the designer expect the structures develop during a severe earthquake, and also are consistent with values of response reduction factors computed from most refined, and consequently time-consuming procedures, applied in recent works.

Additional studies should be performed to verify the applicability of the proposed methodology to different structural typologies, or to irregular in elevation buildings.



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